Evidence for a bimodal distribution in human communication

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Interacting human activities underlie the patterns of many social, technological, and economic phenomena. Here we present clear empirical evidence from Short Message correspondence that observed human actions are the result of the interplay of three basic ingredients: Poisson initiation of tasks and decision making for task execution in individual humans as well as interaction among individuals. This interplay leads to new types of interevent time distribution, neither completely Poisson nor power-law, but a bimodal combination of them. We show that the events can be separated into independent bursts which are generated by frequent mutual interactions in short times following random initiations of communications in longer times by the individuals. We introduce a minimal model of two interacting priority queues incorporating the three basic ingredients which fits well the distributions using the parameters extracted from the empirical data. The model can also embrace a range of realistic social interacting systems such as e-mail and letter communications when taking the time scale of processing into account. Our findings provide insight into various human activities both at the individual and network level. Our analysis and modeling of bimodal activity in human communication from the viewpoint of the interplay between processes of different time scales is likely to shed light on bimodal phenomena in other complex systems, such as interevent times in earthquakes, rainfall, forest fire, and economic systems, etc.

human dynamics | Poisson process | power-law | priority-queue | waiting time

Humans participate in various activities every day in an apparently random manner. By assuming that human actions are Poisson processes (1, 2) in which independent events occur at a constant rate λ and the interevent time τ between two consecutive actions of an individual follows an exponential distribution $P(\tau) = \lambda e^{-\lambda\tau}$, one could perform a quantitative analysis of collective social activities as diverse as disease spreading, emergency response, or resource allocation, in particular phone line availability or bandwidth allocation in the case of Internet or Web use.

Recent evidence from various deliberate human activity patterns, such as e-mail and letter communications and Web surfing, has shown that human activities are nonPoissonian (3–14), with bursts of frequent actions separated by long periods of inactivity, leading to power-law heavy tails in the distributions of interevent time (e.g., interval between sending two consecutive e-mails) or waiting times (e.g., the interval between receiving and replying to an e-mail), $P(\tau) \propto \tau^{-\gamma}$. This nonPoissonian activity should significantly change the quantitative understanding of collective social dynamics, especially when taking into account complex network structures in social interactions (15–17), if those observed nonPoisson activities are solely the behavior of individual agents. Several mechanisms proposed to explain the origin of bursts and heavy tails, including priority-queuing processes driven by human decision making (3, 5, 8, 9, 13), Poisson processes modulated by circadian and weekly cycles (10, 11), adaptive interests (13, 18), and preferential linking (13), have mainly focused on separated individuals. While the power-law waiting time has been regarded as the result of the priority-queuing mechanism of decision making in individuals (3–7), the interevent time of a certain type of activity of an individual, such as the interval between sending two consecutive e-mails, is influenced by the actions of this agent and the other communication partners. The impact of interaction between individuals on human dynamics is, however, still poorly understood.

We can distinguish at least two types of communications: (*i*) initiation by the individual and (*ii*) response to other interacting individuals. Therefore, to distinguish, when possible, what are the properties of separated individuals and what are the consequences of the interactions among individuals, is of paramount importance to elucidate the challenging problem of mutual interplay between individual and collective human dynamics. In particular, are there Poisson processes at all in individual activity, and how do they express themselves when interacting with the decision-making mechanism of individuals and the interaction among individuals? Unfortunately, previously examined data often do not allow us to evaluate precisely both the waiting times and the interevent times, and a detailed analysis of the relationship between individual and collective human activities is still lacking apart of some simple models of coupled priority-queues (19, 20).

Here we address this important problem from both data analysis and modeling. The system we consider is Short Message (SM) correspondence, one of the most frequently used communication systems in modern society. Usually, people can only send e-mails when sitting before the computer. In contrast, people can send and receive SMs almost any time and anywhere. The time required to composite a SM is usually much shorter than other tasks, such as writing an e-mail or letter, making it quite possible to get a prompt response. But it is also flexible, that a SM can be totally ignored with no response given or can be put onto a waiting list as a task with lower priority. These features imply a nontrivial interplay between the activity of single individuals and the interaction with the network neighbors in SMs communication. The system thus provides a very attractive proxy for studying the interaction of human activity. Here we show that this interaction will lead to new types of human activity pattern. The interevent time distribution is a bimodal combination of Poisson and power-law. We demonstrate that the events can be separated into independent bursts; the Poisson and the power-law distributions can be associated to different modes of communication, namely, random initiation of bursts and frequent mutual communication within the bursts, respectively. We propose a minimal

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model incorporating these ingredients with a decision-making mechanism which clearly explains the empirical observations.

Interestingly, a bimodal distribution of interevent time seems quite universal in a wide range of complex systems, including human dialogue (21), trading (22), and financial activity (23) in social systems, but also tsunami (24), rainfall (25), forest fires (26), earthquakes (27, 28), and neuronal avalanches (29), etc. in nature. Here we show that in human communication, bimodal pattern can be attributed to the interplay of various processes at different time scales. Such an approach could shed light on various other bimodal phenomena as well.

Empirical Patterns

We study a database of SMs records from three different companies over a month period (see data description in *Materials and Methods*). While the degree, the number of partners of a user can be quite heterogeneous in SMs networks (30), we have found that many users mainly have heavy communication with just one of their friends. In particular, about 50% of the users sent more than 90% of the messages to one partner (see Fig. S1). Therefore in this work we will mainly focus on such pairs of users, with a typical one shown in Fig. 1. At a first glance, the burst-silence patterns in the individuals (Fig. 1*A*) are similar to many other human activities (3–9). However, the distributions $P(\tau)$ of the interevent time τ, the interval between *sending* two consecutive messages, are bimodal rather than power-law (Fig. 1 *C* and *D*): they are power-law in the range of 2–20 min, followed by an exponential tail extending to 5–6 h, which can be well described as:

$$P(\tau)=\begin{cases}\tau^{-\gamma}, & \tau<\tau_0\\ e^{-\beta\tau}, & \tau>\tau_0\end{cases}. \quad [1]$$

In this bimodal distribution, the exponential tail is connected to the power-law with a hump well above the straight line extrapolation of the power-law. It is important that this feature is significantly different from the usually truncated power-law with the form $\tau^{-\gamma}e^{-\beta\tau}$, where the exponential tail is below the straight line of the power-law and is often considered as finite size effects (5). Note that in a recent report of SM statistics (31), the distributions have been regarded as power-law for the tails also, without paying special attention to the humps and the underlying mechanisms. We can see that the burst-silence patterns of the two users appear to be synchronized (Fig. 1*A*). A clear pattern of sending-response is observed by alternating colors when we join the events of both users (Fig. 1*B*), and we obtain the waiting time τ_w between two consecutive events marked with different colors. Similar to the interevent time τ, the distributions of τ_w also display pronounced bimodal features (Fig. 1 *E* and *F*), in contrast to the prediction of power-law tails of waiting times from the priority-queuing mechanism (3, 5).

The bimodal feature of the distribution is found to be general (see Fig. S2), including those users with many active partners. The exponents γ, γ_w, and β, β_w differ from user to user, with γ centered around 1.5, γ_w around 2.0, and β and β_w around 3.0×10^{-4} (see Fig. S3).

These results are significantly different from previous observations of power-law heavy tails in other human dynamics, such as e-mail communication. The clearly distinguished distributions at

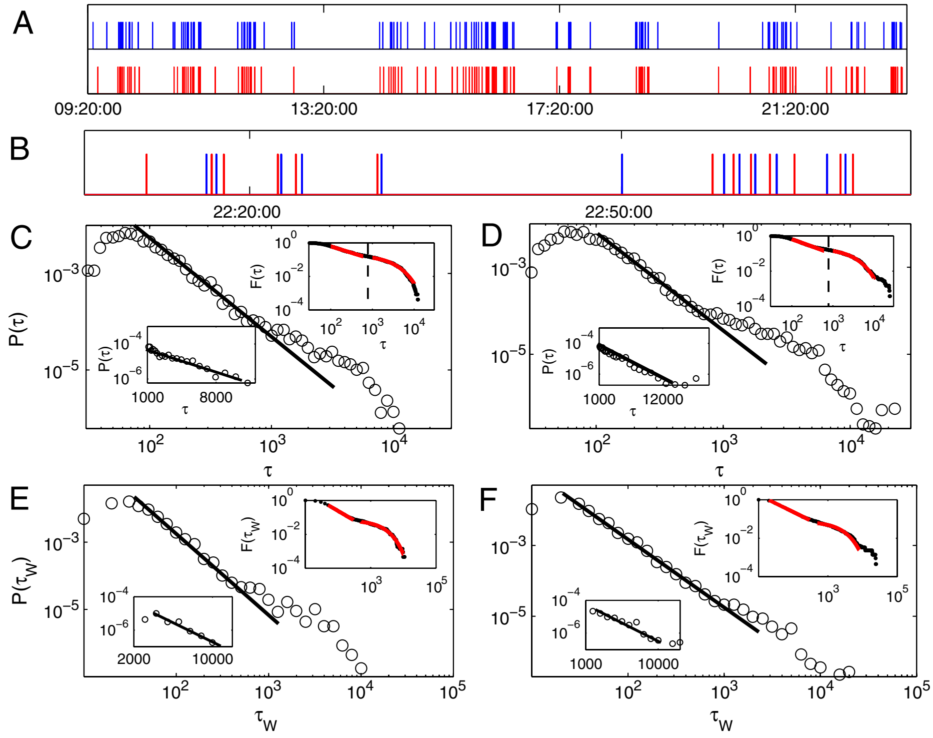

Fig. 1. Typical patterns of SMs activity of a pair of users. The users send more than 95% of the messages to each other. (*A*) Succession of events by user A (blue) and B (red). The horizontal axis denotes time (in 1 s) and each vertical line corresponds to an event of sending an SM. (*B*) An enlargement of a short period where the events of A (blue) and B (red) are put together, showing clearly a sending-response pattern by the alternating blue and red colors. The interval between two consecutive lines with the same color is the interevent time τ and that between two consecutive lines with different colors is the waiting time τ_w. (*C*) and (*D*) are the distributions $P(\tau)$ of the interevent times for the users A and B, respectively. $P(\tau)$ is binned in the log-log scale. The upper inset displays the corresponding accumulative distribution $F(\tau)$. The vertical dotted line indicates $\tau_0 = 780$, which is used to separate the event sequence into independent bursts (see *Materials and Methods* and *SI Text*). The lower inset shows the exponential tails of $P(\tau)$ in the linear-log plot. The straight lines are the power-law and exponential fitting functions, which are correspondingly shown by the red line and red curve in the upper inset. The exponents are: $\gamma_A = 1.79\pm0.01$, $\beta_A = (3.78\pm0.02)\times10^{-4}$ and $\gamma_B = 1.93\pm0.05$, $\beta_A = (3.90\pm0.03)\times10^{-4}$. (*E*) and (*F*) as (*C*) and (*D*), but for the distributions $P(\tau_w)$ of the waiting times τ_w. The exponents are $\gamma_{wA} = 2.12\pm0.01$, $\beta_{wA} = (4.34\pm0.04)\times10^{-4}$ and $\gamma_{wB} = 1.90\pm0.02$, $\beta_{wB} = (3.63\pm0.03)\times10^{-4}$. All the exponents in this work are obtained by the least square method.

small and large intervals imply that there are different processes underlying the observed patterns. Fig. 1*B* shows that a burst is initiated by one of the users, which is then followed by frequent mutual communications. SMs or e-mails suggest that quite likely the initiation of communication over a topic could require a few dense mutual responses. The pronounced exponential tails in the distributions imply that the initiation of communication of the two users could be regarded as independent Poisson processes, which is consistent with the intuition of initiating relatively independent topics of communications in a random manner.

Indeed, we can heuristically separate the events into independent bursts with a crossover time τ_0 even though we have no access to the contents of the communication (see *Materials and Methods* and more details in *SI Text*). Basically, two consecutive messages are considered to be in a burst if the interval $\tau \le \tau_0$ and are regarded as correlated passivity messages, while those messages leading the bursts are regarded as the initiative messages. Firstly, we can take τ_0 somewhat arbitrarily around the crossover between the power-law and exponential parts in the distribution $P(\tau)$ and separate the event sequence into bursts. We can identify the number of bursts whose ith message is sent by user A or B, and represents them by bars with different colors (Fig. 2*A*). The decaying of the bars contains the information of the response probabilities P_A and P_B of the two users A and B to the other, which is very insensitive to τ_0 (see Fig. S4 and *SI Text*). Secondly, we obtain the rates λ_A and λ_B of initiating communications by the user A and B, respectively. Here we assume that the initiations of communication by the two users are independent Poisson processes described by the relationships $\delta_A = \lambda_A + P_A\lambda_B$ and $\delta_B = \lambda_B + P_B\lambda_A$, where δ_A is the rate of bursts in the event sequence of user A, including those initiated by A himself (λ_A) and the response to the initiation of B (λ_B) with the probability P_A, and similarly for δ_B. Now a special τ_0 is chosen such that the separated bursts are best described by independent Poisson processes, quantified by a minimal deviation from the above relationships (Fig. 2*B*). With τ_0 chosen, λ_A, λ_B are also determined, see more details in *SI Text*. Poisson processes of initiation of communication are confirmed by the exponential distribution of the corresponding intervals (Fig. 3*A*). The size n_b of a burst, the number of messages sent by a user in the burst, is determined by the response probabilities P_A and P_B. When user A sends a message, B replies with probability P_B and A sends back again with probability P_A. Thus in one individual the probability of sending another message after the previous one is $P = P_AP_B$. Consequently, the probability to send messages in a burst by one individual is $\prod(n_b) \propto (P_AP_B)^{n_b}$, which predicts precisely that the distribution of n_b follows an exponential function (Fig. 3*B*), with the average size estimated as $\bar{n}_b \approx 1/(1 - P_AP_B)$ for both users. The waiting time τ_w only considering the mutual communication within the bursts very nicely follows power-law distributions (Fig. 3 *C* and *D*), suggesting that the mechanism of priority-queuing and decision making is involved in SM communication. The exponential tails in the waiting time distribution $P(\tau_w)$ (Fig. 1 *E* and *F*) are naturally removed, since the last message of a burst and the first message of the next burst by different users that generate these long intervals are no longer considered as a sending-response pair, but as independent actions.

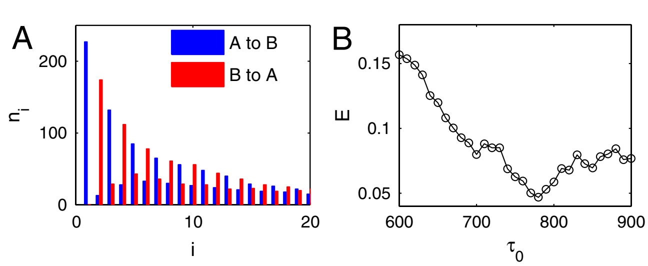

Fig. 2. Separation of bursts and estimation of parameters from data. (*A*) Communication patterns within the separated bursts (obtained at certain τ_0) that are initiated by user A. The index *i* denotes the position of the message in a burst, and the height of the bar (n_i) is the number of bursts having a message at position *i* by user A (blue) or user B (red). (*B*) Relative error $E(\tau_0)$ (see Eq. S2 for definition) displays a minimum where the initiations of bursts in the two users are best approximated by independent Poisson random processes.

Model

These empirical results provide clear evidence that there are three important ingredients in human communication dynamics: independent random Poisson processes to initiate the communication, decision making based on a priority-queuing mechanism, and the interaction among individuals. Here we propose a model of interacting priority queues to obtain more insight into the interplay of these ingredients.

The investigation of human interaction, in particular its effects on the waiting time patterns, started only very recently in models by coupling the priority queues proposed in (3). It has been shown in (19, 20) that interaction between priority queues can change the exponent of the power-law distribution of the waiting time. The priority queues and the schemes of interaction in these models, however, are highly simplifed and could be reasonable only for some special interaction processes in human society. In particular, a list of two tasks of different types, interacting (I) and noninteracting (O), is considered. Two types of interaction schemes are proposed: (*i*) AND-types where the interaction occurs when two individuals pick up at the same time the interacting I-tasks. This scheme could be used to describe common activities such as a meeting. (*ii*) OR-types where the execution of the I-task by one individual will force the other interacting individuals to execute the I-task also, overriding the original priority of the I-task in the waiting list. This OR-protocol of interaction is reasonable for activities such as a phone call. In spite of these preliminary theoretical analyses, the interplay between the individual human activity and the communication among them is still

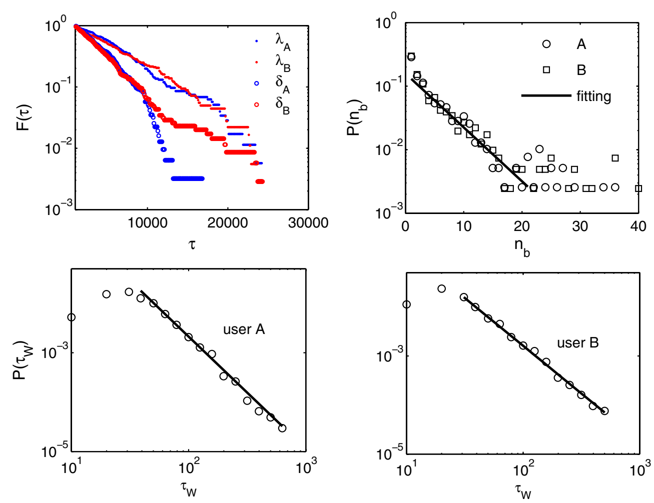

Fig. 3. Separation of the initiative and passivity messages with the most suitable τ_0. (*A*) Accumulative distributions $F(\tau)$ for the interval between two consecutive bursts that are initiated by the same user A (with rate λ_A) and the interval between two consecutive bursts that can be either initiated by A or the response to an initiative message of B (with rate δ_A). These rates of the two users satisfy the relationships $\delta_A = \lambda_A + P_A\lambda_B$ and $\delta_B = \lambda_B + P_B\lambda_A$, implying that the initiation of communications in the two users are independent Poisson processes. (*B*) The distribution of the size n_b of the separated bursts, i.e., the number of messages sent by a user within a burst. The solid line is the exponential fitting $(P_AP_B)^{n_b}$. (*C*, *D*) Distributions $P(\tau_w)$ of the waiting time τ_w obtained only from the messages within the separated bursts, for the user A and B, respectively. The solid lines are the power-law fitting with $\gamma_{wA} = 2.05 \pm 0.01$, $\gamma_{wB} = 1.89 \pm 0.01$.

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largely unexplored, especially, little empirical evidence has been collected.

The model we propose here differs significantly from these previous models of interacting queues. When considering two users as motivated by the empirical observation, it is a minimal model that incorporates the three basic ingredients we observed in the data. The model consists of two main parts: *(i) Priority queuing of tasks of individuals*. A list of tasks are executed one by one with the probability $\prod = x^{\alpha}$, where the random number $x \in (0,1)$ is the priority of the task and α is a tuneable parameter that controls the power-law exponent γ_w in the waiting times (3). This standard model of priority queues is extended in several ways. (1) We introduce a time scale in terms of the processing time t_p and tasks are removed and added to the list every t_p seconds. (2) We distinguish interacting tasks (I-tasks) from the other tasks (O-tasks), similar to (19, 20); and (3) the I-tasks are added to the task list randomly with a small rate $\lambda_p = \lambda t_p$ at each processing step to incorporate the Poisson initiation of tasks observed in the data. *(ii) The interaction between individuals*. This interaction occurs when agent A (B) executes an I-task, which will add an I-task to the list of B (A) with a probability $P_B(P_A)$, i.e., the response rate of B (A). All the I-tasks randomly initiated by an individual and responding to the other will be put onto the waiting list with a random priority x, competing for the execution with the O-tasks. There are three important parameters for each user, λ_i, α_i, and $P_i(i = A,B)$, related to the Poisson process, decision making, and interaction, respectively. More details of the model are presented in *SI Text*.

The model well reproduces all empirical observations with the introduction of the time scale t_p. Note that previous analysis and modeling of e-mail communication took the sampling unit (1 s) just as the unit of actions (3, 5), which is obviously not realistic. The processing times vary for different tasks, but for simplicity we assume it takes t_p seconds to finish each action. The rates for the users to add a new I-task (SMs) in the new time scale is then $t_p\lambda_i$. We simulate the model using the parameters λ_A, λ_B, P_A, and P_B extracted from the data by separating the events into independent bursts. For the parameters α_i used in priority queues, we take $\alpha_i = 1/(\gamma_{wi} - 1)$, where γ_w is the exponent in the power-law distribution of the waiting time τ_w within the bursts in the data (Fig. 3 *C* and *D*). This relation is based on the theoretical formula $\gamma_w = 1 + 1/\alpha$ developed in (3) for this priority-queue model. We simulate the model with different t_p and monitor the relative difference E between the cumulative distributions $F(\tau)$ of the interevent times from the model and the data (see *SI Text*). E has a minimum at $t_p \sim 10$, where the model fits well the distributions of the interevent and waiting times from the data (Fig. 4).

In the following we present a more detailed analysis of the model in order to understand the bimodal interevent time distributions, mainly focusing on the effect of interaction between individuals. Without loss of generality and for simplicity of discussion, we assume that the parameters of the two queues are the same, in particular, $P_A = P_B = P_1$. We also take $t_p = 1$ for the simulations of the model below.

Fig. 5 shows the distribution of interevent time τ for various P_1 when the other two parameters λ and α are fixed. In the extreme case $P_1 = 1$, the process happens as follows: A sends a message, B receives it and waits for a time τ_{wB} to reply to A, and then A waits for a time τ_{wA} to send back again. The time interval between sending two SMs by A (or B), i.e., the interevent time, is $\tau = \tau_{wA} + \tau_{wB}$. Here each of the priority-queue of A or B is the same as the original model (3) where the waiting time is a power-law $P(\tau_{wi}) \propto \tau_{wi}^{-\gamma_{wi}}$. The distribution of the interevent time τ as a sum of the two queues is also a power-law, taking the form $P(\tau) = \tau^{-\gamma_{\min}}$, where $\gamma_{\min}$ is the smaller value of the exponents γ_{wA} and γ_{wB} in the queues A and B (32). Here in our discussion, the two queues are identical, so that $P(\tau) \propto \tau^{-\gamma}$ ($\gamma = \gamma_w = 1 + 1/\alpha$). Since the I-tasks due to mutual communication are created

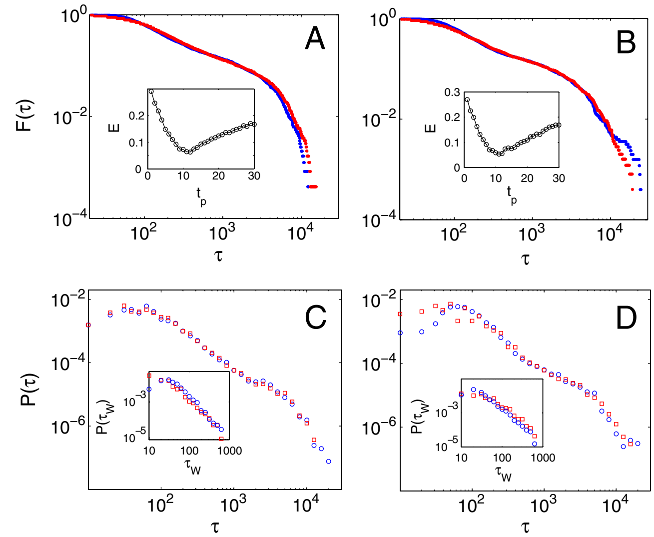

Fig. 4. Fitting of the model (red) to empirical data (blue). The model is simulated for various processing time t_p, using all the other parameters obtained from the data, to generate the same number of events as in the data. The relative difference between the accumulative distributions $F(\tau)$ of the interevent times τ in the model and data, is obtained as a function of t_p (averaging over 10 realizations of independent model simulations, insets of (*A*, *B*)). E is minimal at $t_p = 10$ for both users, yielding very accurate fitting of $F(\tau)$ (*A*, *B*), $P(\tau)$ (*C*, *D*) and $P(\tau_w)$ (insets of (*C*, *D*)), except for a few points with the minimal and maximal intervals mainly due to finite size fluctuations.

with a much higher probability than the Poisson rate λ, the pattern of the interevent time will be dominated by the power-law, as seen clearly in Fig. 5. Note that the case of $P_1 = 1$ in our model corresponds to a model previously proposed to explain the power-law interevent times in e-mail communication from the power-law waiting times due to priority-queuing mechanism (5). In that model it was assumed that e-mail communication is the process that A sends an e-mail to B as a response to an e-mail B sends to A and vice versa in an endless manner (5). This model is in contrast with the evident facts about e-mail communication where we do not reply to every e-mail (thus $P_1 < 1$), and we also initiate independent communications in addition to passive responses.

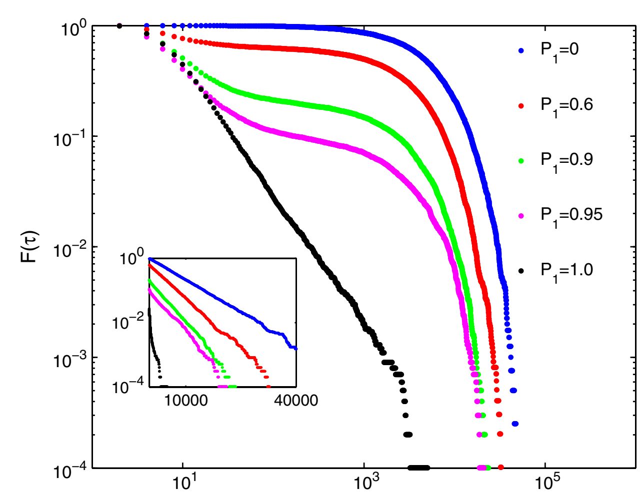

Fig. 5. Effect of interaction on human activity patterns in the model. The cumulative distribution $F(\tau)$ of the interevent times obtained at various response rates P_1. The other parameters are fixed as $\lambda = 1.5 \times 10^{-4}$, $\alpha = 1.0$, and $t_p = 1$. The inset shows the exponential tails in the linear-log plot.

It is important to emphasize that the activity patterns are very sensitive to P_1. As seen in Fig. 5, when P_1 is only slightly smaller than 1.0 (e.g., $P_1 = 0.95$), the distribution is no longer a complete power-law, but clearly bimodal with a pronounced exponential tail. This behavior happens because the frequent mutual communication will be terminated: the probability to get bursts of large size decreases exponentially ($\prod(n_b) \propto P_1^{2n_b}$). This result means that the mechanism of mutual response as proposed in (5) cannot explain the power-law behavior in the e-mail communication when P_1 is not exactly 1.0. As discussed in more detail in the *SI Text*, our model with the processing time t_p provides an alternative, more natural explanation which allows us to generate a power-law distribution with $P_1 < 1$ (see Fig. S5).

A value of P_1 close to but less than 1.0 is important for a pronounced bimodal distribution. The bursts have an average size $\bar{n}_b = 1/(1 - P_1^2)$. Here a large number of SMs are replied, but they are put onto the waiting list with a random priority x, and the interevent time for these events follows a power-law distribution $P(\tau) \propto \tau^{-\gamma}$. The power-law distribution is cut off by the finite number of messages $\bar{n}_b$ within bursts, leading to a crossover waiting time τ_0. $\bar{n}_b$ and τ_0 are related as $\int_{\tau_{0p}}^{\infty} P(\tau)d\tau = 1/\bar{n}_b$, where $\tau_{0p} = \tau_0/t_p$ is the cut-off in the unit of processing step. Putting $P(\tau) \propto \tau^{-\gamma}$, we get

$$\tau_0 \propto t_p (\bar{n}_b)^{1/(\gamma-1)} \propto t_p (1 - P_1^2)^{-1/(\gamma-1)}. \qquad [2]$$

Thus the crossover time τ_0 is on average larger if P_1 is closer to 1.0 because there will be a larger number of SMs in a burst. As a result we observe a regime of power-law distribution of the interevent time: there are many more short and intermediate intervals than we can expect from the Poisson processes only. The bursts and the power-law regime will not be clearly observable when P_1 becomes smaller, since $\bar{n}_b = 1/(1 - P_1^2)$ becomes too small and the distribution is dominated by the exponential function. This situation happens already for relatively large response rates, e.g., when $P_1 = 0.8$ we have $\bar{n}_b < 3$.

As seen from the inset of Fig. 5, the interevent time distributions display pronounced exponential tails when $\lambda \ll 1$, with the exponent β depending on the value of $P_1 < 1$. This result can be understood as follows: (*i*) The two users initiate communications independently with the rate λ, and respond to each other with the probability P_1. Consequently in the event sequence of an individual, we observe independent bursts either initiated by the individual or the response to the other with the rate $\delta = \lambda + P_1\lambda$, and the interval between the first message of two consecutive bursts is τ_δ. In the interevent distribution $P(\tau)$, the tail corresponds to long intervals between the last message of one burst and the first message of the next burst, $\tau = \tau_\delta - \tau_b$, where τ_b is the total time spent in the first burst. The interval within the burst follows a power-law distribution, and we have $\tau_b = n_b \int^{\tau_0} P(\tau)\tau d\tau \sim \tau_0$. As a result, for those long intervals we get $\tau \approx \tau_\delta$, corresponding to the exponential tails with the exponent $\beta \approx \delta = \lambda + P\lambda$ when $P_1 < 1$.

The analysis of the model reveals the importance of the interplay among three ingredients in human communication patterns. The communication cannot continue without random initiation of I-tasks; and if all the messages are replied ($P_1 = 1$) after the first initiation, the communication almost cannot stop to allow the initiation of new I-tasks. Such situations are not realistic. If there is not the ingredient of interaction ($P_1 = 0$), each individual only sends SMs initiated randomly without getting a response. For these randomly initiated I-tasks, the time spent on the waiting list is small compared to the average Poisson interval $1/\lambda$ because the newly added task has higher priority on average compared to the other tasks still on the waiting list. As a result, the interevent time is close to the Poisson distribution (Fig. 5). Finally, if there is not a mechanism of priority-queuing, e.g., the tasks are randomly selected for execution when $\alpha = 0$, we cannot expect a regime of power-law interevent time.

In summary, the model explains the empirical observations in the following way. (*i*) The response rates P_A and P_B control the average burst size $\bar{n}_b \approx 1/(1 - P_A P_B)$. (*ii*) The power-law waiting times τ_{wA}, τ_{wB} due to the priority-queuing mechanism lead to power-law interevent times in the bursts ($\tau = \tau_{wA} + \tau_{wB}$), with the exponent $\gamma = \min(\gamma_{wA}, \gamma_{wB})$ (32). (*iii*) However, this power-law distribution cannot extend to large intervals, because there is a cut-off τ_0 due to the finite event size $\bar{n}_b$ of the bursts, $\tau_0 \sim t_p(1 - P_A P_B)^{-1/(\gamma-1)}$. (*iv*) The distribution of τ displays a pronounced bimodal feature if $\tau_0 \ll 1/\beta$, the characteristic interval of the Poisson random actions.

Discussion

We emphasize that the three basic ingredients investigated in this work are common in many other types of human communication activity, such as instant chat in the internet (e.g., MSN, Google-talk, and Skype, etc.), e-mail and letter communications, and human dialogue, etc. Our model is readily applicable to these situations. For instance, in e-mail and letter communications, there are also passivity events of consecutive exchanges and random initiation of the communication and clearly not all the e-mails or letters receive a reply. In letter communication, the waiting time distributions without separating the initiative and passivity messages do show clear bimodal features with humps in the exponential tails (4, 5), similar to Fig. 1 *E* and *F* in SMs. The interevent time is found to follow an exponential distribution (5), which can be explained in our model by a relatively small response rate (see *SI Text*). A close inspection of previously published results of e-mails does indicate the bimodal nature of the distributions of interevent times (5), but not as pronounced as in SMs. In our framework, the distribution will shift gradually from bimodal to a truncated power-law $\tau^{-\gamma}e^{-\beta\tau}$ when the cut-off time τ_0 becomes larger and comparable to the characteristic interval of the Poisson initiation (see Fig. S5). Larger τ_0 could be attributed to longer processing time t_p (and a smaller power-law exponent γ as well), which is consistent with the e-mail communication.

Our findings reveal that there is a generic Poisson process in individual human behavior which is connected to the power-law-like bursts through the interaction with other individuals, resulting in the interplay between the cut-off time τ_0 and the characteristic Poisson interval $1/\beta$ which are generally influenced by the network topology and the processing time t_p in various human activities. This picture has significantly changed the current competing views of human activity, either following Poisson or power-law statistics. Our findings open a new perspective in understanding human behavior both at the individual and network level which is of utmost importance in areas as diverse as rumor and disease spreading, resource allocation and emergency response, economics, and recommendation systems (33–36), etc. For example, treating the events as independent bursts would allow quantitative analysis of phone line availability and bandwidth allocation in the case of Internet or Web use, which should be significantly different from the assumption of power-law tails which allow very long silent periods.

Bimodal distributions are not limited to human communications, but are also typical in other interacting social systems, such as trading (22). With suitable modification, our model could be applied to understand the bimodal interevent distribution of these systems.

Bimodal interevent times are also widely observed in diverse natural systems ranging from rainfall to earthquakes and neuronal avalanches (24–29). The method of separation of the events into independent bursts in this work should be useful for the analysis of these bimodal natural phenomena. The origin of the bimodal interevent times varies in different natural systems, but a

Table 1. Information of the data

Name of the company	The total number of the records	The number of the users	The number of the active users *
A	548, 182	44, 430	9, 567
B	643, 502	72, 146	12, 162
C	398, 185	31, 096	7, 727

*Who sends more than five SMs and receives more than five SMs is considered as an active user

common and general feature is that the distinct distributions could be associated to processes of different temporal or/and spatial scales. For example, in earthquakes, there is independent seeding (background) activity at longer times that triggers correlated aftershocks in short times due to time-dependent relaxation of the crust (28). Therefore, a more comprehensive understanding of various complex systems would require the investigation of the interplay of various processes competing at different spatial/temporal scales, as we demonstrated here for the human communication activity.

Materials and Methods

Data Description. The data investigated in this work were obtained from a mobile phone operator. The data include three charging accountant bills from three companies over 1 mo period. Each record comprises a sender mobile phone number, a recipient mobile phone number and a time stamp with a precision of 1 s. The detailed information about the data is listed in Table 1.

For the purpose of retaining customer anonymity, each subscription is identified by a surrogate key such that it is not possible to recover the actual phone numbers from it. There is no other information available for identifying or locating customers, which guarantees that their privacy is fully respected.

The interevent time in our analysis is the time interval between *sending* two consecutive messages. For active users with at least several messages per day, the longest waiting time during a day is limited to 5–6 h, on average shorter than the time interval between the last message of 1 d and the first message of the next day (8–9 h). We thus exclude the time intervals crossing 2 d from the analysis, as they have negligible effects on our results. Note that such time intervals associated with a sleep break may not be so neatly separated for inactive users, and for many other human activity occurring at slower scales.

Separating Events into Independent Bursts. Using a crossover time τ_0, we can divide the events into bursts in which frequent communications are separated with an interval $\tau < \tau_0$, and consequently determine all the messages initiating the bursts. From such bursts obtained using different τ_0, we can reliably estimate the response rates P_A and P_B for the two users A and B, respectively. Finally, the most suitable τ_0 is chosen such that the initiations of communication of the two users are best fitted by two independent Poisson processes, and the rates λ_A and λ_B of the random initiations are determined. Using the waiting time statistics from the separated bursts, we can estimate the parameters α_A and α_B with a formula developed in priority-queuing theory (3). All these empirical parameters are then put into the model of two coupled priority-queues to reproduce the distributions of waiting times and interevent times with a suitable processing time t_p. More details of the methods to obtain empirical parameters, the analysis of the model, and application of the model to describe some other interacting human activity are presented in *SI Text*.

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